CREEP TESTS ON GLASS-REINFORCED PLASTIC PLATES

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A method of designing thin-walled glass-reinforced plastic (GRP) systems with two-way reinforcing for creep was proposed in [1, 2]. The results of an experimental verification of this method by testing square plates in pure torsion and beams in bending are given below.

Conditions of pure torsion can be realized by two methods: a uniformly distributed torque H can be applied at the ends or the specimen can be loaded as shown in Fig. 1, where the load P = 4H [3-5]. In the latter case, pure torsion (pure shear, nonhomogeneous over the thickness) is obtained throughout the plate with the exception of the edge zone. This singularity will, however, be disregarded in the calculations.

The boundary conditions are

$$M_x = 0 \quad \text{at} \quad x = 0,\ x = a; \qquad M_y = 0 \quad \text{at} \quad y = 0,\ y = a, \qquad Q(x, y, t) = 0$$
$$w(0, 0) = 0 \qquad w(a, a) = 0; \qquad w(a, 0) = w(0, a),$$
$$H_{xy} = H(t) \quad \text{at} \quad x = 0,\ x = a; \qquad y = 0,\ y = a \qquad (t = \text{time}) .$$

The deflection is sought in the form

$$w = A(t)\,x^2 + B(t)\,xy + C(t)\,y^2 + C_1(t)\,x + C_2(t)\,y + C_0(t) .$$

The stress-strain relation is postulated in the form

$$\varepsilon_x = \frac{1}{E_1}\sigma_x - \frac{\nu_2}{E_2}\sigma_y, \quad \varepsilon_y = -\frac{\nu_1}{E_1}\sigma_x + \frac{1}{E_2}\sigma_y ,$$
$$\gamma_{xy} = \frac{1}{\mathrm{G}}\tau_{xy} = \frac{1}{G^\circ}\left[\tau_{xy} + \chi\int_0^t \frac{(t-\theta)^\alpha}{\Gamma(1+\alpha)}\tau_{xy}(\theta)\,d\theta\right], \tag{1}$$

where σ_x, σ_y, τ_{xy}, ε_x, ε_x, γ_{xy} are the stresses and engineering strains along the principal axes of anisotropy x and y; G is the instantaneous shear modulus; χ is a dimensional constant; Γ is the gamma function; and G is the creep operator [6].

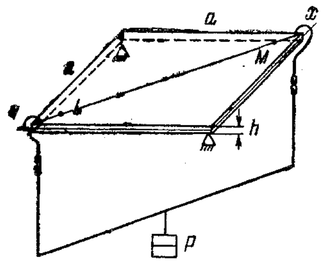

Fig. 1.

The deflection equation and the expressions for the bending and torsional moments are the same as in [1]. By employing the boundary conditions and Eqs. (1), we obtain

$$w = -\frac{6}{h^3}\left[xy - \frac{a}{2}(x+y)\right]\frac{1}{\mathrm{G}}H(t) . \tag{2}$$

Two square plates of GRP AG-4s with identical reinforcing along the x and y axes were tested by the method shown in Fig. 1 at loads varied in steps and a constant temperature of 30°C. The total deflection at the points L (x = 10 mm, y = 230 mm) and M (x = 230 mm and y = 10 mm) was measured with a simple indicator device with 0.01 mm graduations. The plate dimensions were a = 240 mm, h = 5.84 mm for the first plate, and h = 5.75 mm for the second.

Both plates were initially subjected to creep at constant load and then allowed to recover in the unloaded state; when the recovery rate became sufficiently small, the plate was turned over and loaded with the same force, which resulted in a change in the sign of the stress. Hence, the loading schedule was as follows:

$$P = P_0 \quad \text{for} \quad 0 < t < t_1, \qquad P = 0 \quad \text{for} \quad t_1 < t < t_2, \qquad P = -P_0 \quad \text{for} \quad t_2 < t . \tag{3}$$

Since the relation between the stresses and creep strains is practically linear [2], as confirmed by experiments on specimens cut out of the plates after testing and relaxation, the linear memory theory can be verified directly from the curves of variation in deflection at the points L and M. In the first part of schedule (3) the creep curves were approximated by the function

$$w(t) = w^\circ + ft^{0.23} \qquad > t_1$$

Moreover, theoretical curves were plotted by means of the principle of superposition:

$$w(t) = f\,[t^{0.23} - (t - t_1)^{0.23}] \quad \text{for} \quad t_1 < t < t_2$$
$$w(t) = f\,[t^{0.23} - (t - t_1)^{0.23} - (t - t_2)^{0.23}] - w^\circ \quad \text{for} \quad t > t_2 . \tag{4}$$

For the first plate the load changing times and constants were: t_1 = 600 hours, t_2 = 772 hours, P_0 = 6.720 kg, f = = 0.24, w° = 2.92 mm, and for the second: t_1 = 600 hours, t_2 = 672 hours, P_0 = 5.800 kg, f = 0.32, and w° = 3.02 mm. The agreement between the experimental points in Figs. 2 and 3 and the theoretical curves plotted from Eq. (4) is satisfactory.

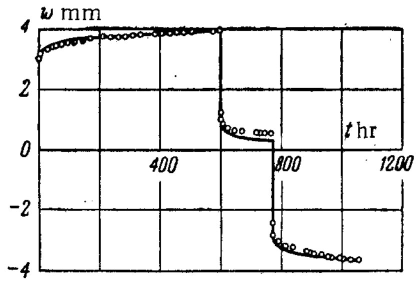

Fig. 2.

A third plate was tested by the same procedure (Fig. 1, h = 6 mm) at a constant load P = 7.550 kg. After one month's relaxation at a test temperature of 30°C, followed by three months at room temperature, it was cut into four beams whose longitudinal axes formed angles φ = 0, 22.5, 45 and 90° with one of the reinforcing directions. These beams (length 200 mm, width b = 25 mm) were then supported on prisms (spacing l = 185 mm) and loaded with a constant load P_1 = 4 kg in the center of the span. Since the superposition principle is fulfilled with a sufficient degree of accuracy, it can be assumed that after relaxation the plate returned practically to its starting state. The experiments carried out at φ = 0, 90 and 45° are regarded as basic; the creep of the beam at φ = 22.5° and the creep of the plate (out of which these beams were cut) in pure torsion are predicted from the same results. The deflections of the beams at φ = 0 and 90° turned out to be close and in the computations were assumed to be identical and equal to the average value.

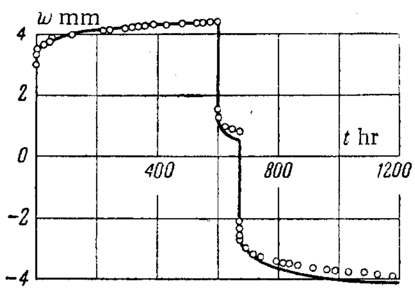

Fig. 3.

The results of the basic experiments (circles) and their approximation on the basis of S. G. Lekhnitskii's hypotheses [2] are shown in Fig. 4. The curve for φ = 45° is derived directly from the experimental data. The values for the deflection at φ = 0° (90°) were approximated by a straight line drawn through the point [t = 30 hours, w = w(30)] parallel to the time axis.

The formula for the deflection of a beam w_φ, cut out at an angle φ, under a constant load can be written as

$$w_\varphi = \frac{P_1 l^3}{4bh^3} E_\varphi^{-1} \cdot 1 = \frac{P_1 l^3}{4bh^3} [\varepsilon_\varphi^{\circ} + \varepsilon_\varphi^{c}(t)] , \quad (5)$$

where E_φ^{-1} is the creep operator in the direction φ [2]; the creep strains described by this operator satisfy the relation

$$E_\varphi^{-1} \cdot 1 = \varepsilon_\varphi = \varepsilon_1 \cos^4 \varphi + \tfrac{1}{4} [\gamma(t) + 2\omega_1] \sin^2 2\varphi + \varepsilon_2 \sin^2 2\varphi , \quad (6)$$

where ε_1 and ε_2 are the strains in the principal directions for unit tensile (compressive) stress; $\gamma(t)$ is the shear strain for unit shear stress along the principal axes x and y; w_1 is the transverse strain for unit tensile (compressive) stress acting along the x axis; ε_φ is the longitudinal strain along an axis forming an angle φ with the x axis for unit tensile (compressive) stress; it is composed of elastic and creep components:

$$\varepsilon_\varphi = \varepsilon_\varphi^{o} + \varepsilon_\varphi^{c} .$$

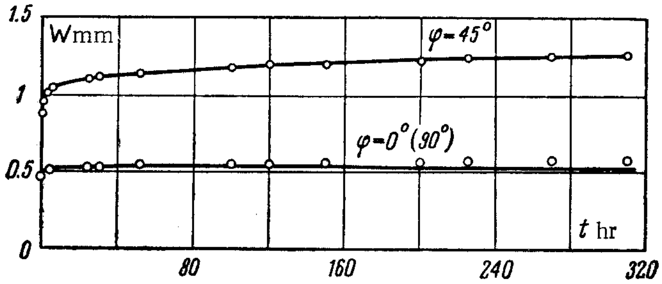

Fig. 4.

In accordance with the accepted approximation there is no creep in the x and y directions, while the strains ε_1 and ε_2 for constant stress are assumed equal to the experimental values at t = 30 hours. The values of the width, thickness and load for the individual beams mentioned above differed somewhat; hence the expressions for each angle are written as follows:

$$\varphi = 0^\circ\ (90^\circ), \qquad w_1 = 1.17 \cdot 10^3 \varepsilon_1 ,$$

$$\varphi = 45^\circ, \qquad w_{45} = 1.20 \cdot 10^3 \varepsilon_{45} = 1.2 \cdot 10^3 [\varepsilon_{45}^{\circ} + \varepsilon_{45}^{c}(t)] ,$$

$$\varphi = 22.5^\circ, \qquad w_{22} = 1.19 \cdot 10^3 \varepsilon_{22} = 1.19 \cdot 10^3 [\overset{\circ}{\varepsilon}_{22} + \varepsilon_{22}^c(t)].$$

For $\varphi = 45°$ and $\varphi = 22.5°$ (with $\varepsilon_1 = \varepsilon_3$) it follows from (6) that

$$\varepsilon_{45} = 0.5\varepsilon_1 + 0.25\,[\gamma(t) + 2\omega_1], \quad \varepsilon_{22} = 0.75\varepsilon_1 + 0.125[\gamma(t) + 2\omega_1]. \tag{7}$$

Therefore

$$\overset{\circ}{\varepsilon}_{22} = 0.5\,(\varepsilon_1 + \overset{\circ}{\varepsilon}_{45}), \quad \varepsilon_{22}^c(t) = 0.5\,\varepsilon_{45}^c(t)\ . \tag{8}$$

Equation (8) enables us to calculate the change in deflection for $\varphi = 22.5°$ from the data for beams with $\varphi = 0°$ (90°) and $\varphi = 45°$. The results of this calculation are represented by the lower curve in Fig. 5 (both the theoretical curve and the experimental points correspond to tripled values of the deflection w_{22}). The agreement is completely satisfactory.

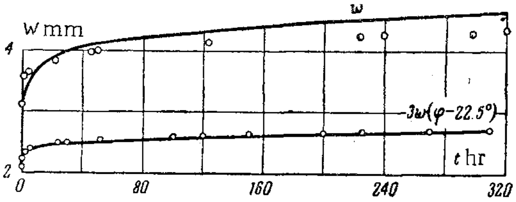

Fig. 5.

The behavior of the starting plate in pure torsion was calculated analogously from the basic experiments ($\varphi = 0$, 90, 45°).

It follows from (7) that

$$\gamma^\circ = 4\varepsilon_{45}{}^\circ - 2\varepsilon_1 - 2\omega_1, \qquad \gamma^c(t) = 4\varepsilon_{45}{}^c(t).$$

In accordance with the experimental data [2], ν_1 can be taken as equal to 0.07; then $w_1 = -\nu_1\varepsilon_1 = -0.07\,\varepsilon_1$. M Moreover, in (2) we put $G^{-1} = \gamma^\circ + \gamma^c(t)$ and numerical values for the coordinates of the point L as well as the force $P_1 = 4H$ and the thickness of the plate. The results of the computation are represented by the upper curve in Fig. 5; its deviation from the experimental points does not exceed 8%.

Hence, it can be concluded that the mathematical model of a glass-reinforced plastic presented in [2] satisfactorily describes the behavior of the actual material.

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